AQA A-Level Mathematics Warmup - Paper 3 2024

Sketch on the same axes $f(x) = 1 + |x|$ and $g(x) = |3 - 2x|$. Solve $f(x) = g(x)$.

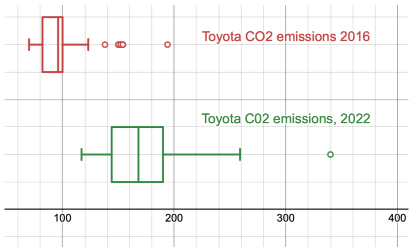
A student claims that $p(n) = n^2 + n + 5$ will generate a prime number for all $n \in \mathbb{N}$. What is the smallest integer counter example to this claim?

When are two events, A and B independent?

Write $3\sin(\theta) + 4\cos(\theta)$ in the form $R\sin(\theta + \alpha)$.

Convert the parametric equations $x(\theta) = 2 + 3\cos(\theta)$ $y(\theta) = -1 + 4\sin(\theta)$ to the Cartesian form.

Find $\displaystyle\int \frac{8x + 10}{2x^2 + 5x + 4}\, dx$



a) The box plots to the left display information about the CO2 emissions of Toyota cars (as shown in the AQA LDS). Compare the emissions in 2016 (top) and 2002.
b) Why can't you make the same conclusion for all cars?

For a normal distribution X, complete the following statements:
1) The points of inflection are ..... standard deviation away from the mean.
2) Total area under the curve is .....
3) ~...... % of values lie within σ of the mean.
4) ~ .... of values lie within 3σ of the mean.
5) $P(X > \mu + a) = P(X < \ldots\ldots)$

A cube of side length $2x$ is expanding . Let A be the surface area and V be the volume. By finding $\frac{dA}{dx}$ and $\frac{dV}{dx}$ show that $\frac{dA}{dt} = \frac{4}{x}$ if $\frac{dV}{dt} = 2$.

Find $\displaystyle\int e^x \sin(x)\, dx x$

Let $X \sim B(20,0.15)$
a) Find $P(X = 2)$
b) Find $P(X \leq 8)$
c) Find $P(2 \leq X < 6)$
d) The expected value
e) The variance

For $0 \leq x \leq 2\pi$ sketch on the same axes $y = \cos(x)$ and $y = \sec(x)$

A company sells 400g cans of beans. Trading standards samples 20 of these and finds a sample mean of 396g. Suppose that the underlying distribution has a standard deviation of 5g. Test at the 1% significance level to see if there is evidence the cans are being produced underweight.

Use the Newton-Raphson method with $x_0 = 1.5$ to find the approximations x_1 and x_2 to the solution of $e^{\sin(x)} + 2x - 4 = 0$.

Find the equation of the tangent to the curve $y = x\sin(x)$ at $x = \frac{\pi}{3}$

A chocolate is selected randomly from a box. The probability of it containing caramel is 0.25. The product of it containing hazelnuts is 0.1. The probability of it containing both caramel and hazelnuts is 0.05. Find the probability that a randomly chosen chocolate contains either nuts or caramel.

Let $X \sim N(170,8)$. Find,
a) $P(X = 168)$
b) $P(X \leq 172)$
c) $P(X \geq 165)$
d) $P(167 \leq X \leq 171)$

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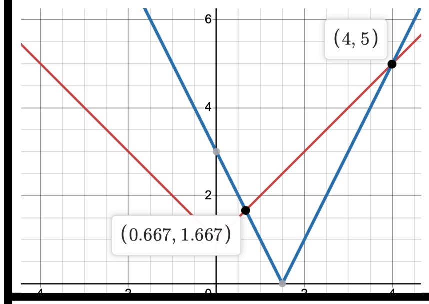


$p(0) = 5$
$p(1) = 7$
$p(2) = 11$
$p(3) = 17$
$p(4) = 25$

And so, $n = 4$ is the smallest counter example.

Events A and B are independent if

$$P(A \mid B) = P(A)$$

Using the addition formula for sine

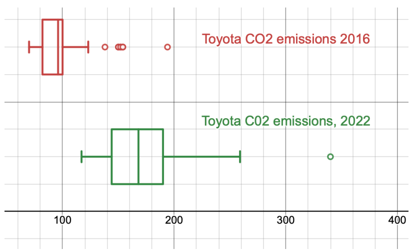
$3\sin(\theta) + 4\cos(\theta) = R\left(\sin(\theta)\cos(\alpha) + \cos(\theta)\sin(\alpha)\right)$

So, $3 = R\cos(\theta)$ and $4 = R\sin(\theta)$. Eliminating θ we see that $R = \sqrt{3^2 + 4^2} = 5$ and $\alpha = \arctan\left(\frac{4}{3}\right)$. Hence, we obtain $5\sin(\theta + 0.9273..)$

Rearranging, $\frac{x-2}{3} = \cos(\theta)$ and $\frac{y+1}{4} = \sin(\theta)$ and so, using $\cos^2(\theta) + \sin^2(\theta) = 1$, we obtain,

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$$

$$2\ln|2x^2 + 5x + 4|$$



a) In general the emissions in 2016 are significantly lower than in 2022 as evidenced by the difference in medians. The IQRs also show that the spread of emissions is lower. There are more outliers in 2016.
b) The emissions for Toyota do not necessarily reflect all registered cars.

1) The points of inflection are 1 standard deviation away from the mean.
2) Total area under the curve is 1.
3) ~.68 % of values lie within σ of the mean.
4) ~ .99.8 of values lie within 3σ of the mean.
5) $P(X > \mu + a) = P(X < \mu - a)$ for any constant a.

$A = 24x^2 \Rightarrow \frac{dA}{dx} = 48x$

$V = 8x^3 \Rightarrow \frac{dV}{dx} = 24x^2$

$\frac{dA}{dt} = 48x\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{1}{12x}$

$\frac{dA}{dt} = \frac{4}{x}$

Using Integration by Parts twice

$I = e^x\sin(x) - \int e^x\cos(x)\,dx$

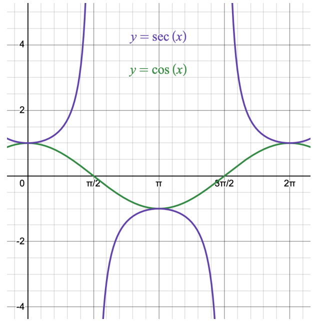
$= e^x\sin(x) - e^x\cos(x) - \int e^x\sin(x)\,dx$

So,

$2I = e^x\sin(x) - e^x\cos(x)$

$\Rightarrow I = \frac{1}{2}\left(e^x\sin(x) - e^x\cos(x)\right)$

a) $P(X = 2) = 0.2293$
b) $P(X \leq 8) = 0.9987$
c) $P(2 \leq X < 6) = P(X \leq 5) - P(X \leq 1)$ so $P(2 \leq X < 6) = 0.7571$
d) $20 \times 0.15 = 3$
e) $20 \times 0.15 \times 0.85 = 2.55$



Let $X =$ weight of a can of beans. Then $X \sim N(400, 5^2)$. The sample means are distributed

$$\overline{X} \sim N\left(400, \frac{5^2}{20}\right).$$

Let $H_o : \mu = 400$ and $H_1 : \mu < 400$.

We have a $p-$value $= P(\overline{X} \leq 396) = 0.002$. Since $0.0002 \leq 0.01$ there is sufficient evidence to reject H_0 and conclude cans are being sold underweight.

Let $f(x) = e^{\sin(x)} + 2x - 4$ then $f'(x) = e^{\sin(x)}\cos(x) + 2$. The N-R iteration is

$$x_{n+1} = x_n - \frac{e^{\sin(x_n)} + 2x_n - 4}{e^{\sin(x_n)}\cos(x_n) + 2}$$

$x_1 = 0.7191445372$ and $x_2 = 0.9013623019$.

By the product rule, $\frac{dy}{dx} = \sin(x) + x\cos(x)$.
At $x = \frac{\pi}{3}$ the gradient of the tangent is $\frac{3\sqrt{3} + \pi}{6}$ and it passes through $\left(\frac{\pi}{3}, \frac{\pi}{2\sqrt{3}}\right)$ giving the equation of the tangent.

$$y = \left(\frac{\sqrt{3}}{2} + \frac{\pi}{6}\right)x - \frac{1}{3}\left(\frac{\sqrt{3}}{2} + \frac{\pi}{6}\right) + \frac{\pi}{2\sqrt{3}}$$

Let N denote containing nuts and C denote containing caramel. Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, we have

$P(N \cup C) = 0.25 + 0.1 - 0.05$
$= 0.3$

a) $P(X = 168) = 0$
b) $P(X \leq 172) = 0.5793$
c) $P(X \geq 165) = 1 - 0.6915$
$= 0.3085$
d) $P(167 \leq X \leq 171) = 0.1960$