Dr Bennison Maths

MATHEMATICS EDUCATION EXPERT

AQA AS-Level Further Maths 2026
Paper 2S (Statistics)

Do not turn over the page until instructed to do so.

This assessment is out of 40 marks and you will be given 45 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 40

1 The discrete random variable X has probability distribution

$$f(x) = \begin{cases} 0.2 & x = 2 \\ 0.35 & x = 4 \\ 0.4 & x = 6 \\ 0.05 & x = 8 \end{cases}$$

What is the mode of X?

~~0.4~~ 6 ~~0.05~~ 4 0.4 0.05

[1 mark]

2 The continuous random variable X has $\mathrm{E}(X) = 3$.

The discrete random variable Y is independent of X and such that $\mathrm{E}(Y) = 2$.

What is $\mathrm{E}(X - Y)$

1 5 6 13

[1 mark]

3 A binomial hypothesis test is carried out at the $5\,\%$ level of significance with the hypotheses

$$H_0: p = 0.4$$
$$H_1: p < 0.4$$

A sample of size 40 was used to carry out the test.

Find the probability that a Type I error was made.

Circle your answer.

$1.6\,\%$ $\quad$ $3.8\,\%$ $\quad$ $7.1\,\%$ $\quad$ $3.5\,\%$

[1 mark]

4 A random variable X has a normal distribution with known variance 6.8.

A random sample of size n is taken from X and the sample mean is found to be 56.2.

A $96\ \%$ confidence interval is constructed.

a) Given that the interval is $(55.730,\ 56.670)$ find the value of n.

[4 marks]

For a 96% confidence interval $z = 2.05375$

Then

$$56.670 - 55.730 = 2 \times 2.05375\sqrt{\frac{6.8}{n}}$$

$$\Rightarrow \sqrt{\frac{6.8}{n}} = \frac{0.94}{2 \times 2.05375}$$

$$\Rightarrow \frac{6.8}{n} = 0.052372169 28$$

$$\Rightarrow n = \frac{6.8}{0.05237216928}$$

$$= 129.839953$$

Hence $n = 130$ is the sample size

b) Tom claims that the true mean is 55.9.
Does the confidence interval support this claim? Give a reason for your answer.

[1 mark]

Yes, as 55.9 lies within the confidence interval.

5 An insurance company models the number broken phone claims it receives in one day by a Poisson distribution with parameter $\lambda = 50$.

a) Suggest why a Poisson distribution could be a suitable model for this situation?

[1 mark]

It is reasonable to assume that the phone calls to the insurance company occur singly in time and are independent of each other

b) Find the probability that the company receives at most 40 claims for broken phone screens on a randomly selected day.

[1 marks]

Let X be the RV = number of broken screen claims per day:
Then $X \sim Po(50)$

$$P(X \leq 40) = 0.0861$$

c) **(i)** Customers can only make a claim by phoning a line that is open for 10 hours per day.

What is the probability of the line receiving 6 calls in a given hour?

[1 mark]

Let Y be the RV = number of claims received by the phone line in an hour:
Then $Y \sim Po\left(\frac{50}{10}\right)$, i.e. $Y \sim Po(5$

$$P(Y=6) = 0.1462$$

6 A random variable, X, has probability density function

$$f(x) = \begin{cases} -kx(x-4) & 0 \leq x < 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

a) Find the value of k.

[2 marks]

Since

$$\int_0^4 -kx(x-4)\,dx = 1$$

$$\Rightarrow -k\int x^2 - 4x\,dx = 1$$

$$\Rightarrow -k\left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_0^4 = 1$$

$$\Rightarrow -k \times \frac{-32}{3} = 1$$

$$k = \frac{3}{32}$$

Hence,

$$f(x) = \begin{cases} -\frac{3}{32}x(x-4) & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

b) Is the median of $f(x)$ greater than the mean of $f(x)$?

Fully justify your answer.

[3 marks]

Median:

$$\int_0^M -\frac{3}{32}x(x-4)\,dx = \frac{1}{2}$$

$$\Rightarrow \quad -\frac{M^3}{32} - \frac{6M^2}{32} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{M^3}{32} - \frac{6M^2}{32} + \frac{1}{2} = 0$$

So $M = 5.464$, $M = 2$, $M = -1.464$

So $M = 2$ as no others in the range.

Mean:

$$E(X) = \int_0^4 -\frac{3}{32}x(x-4)\cancel{x}\,dx$$

$$= 2$$

No, the median is not greater, it is equal.

(ii) Jess records how many calls were received in hour periods for 10 days.

What is the probability that at least 6 of the 10 days had an hour where 6 calls were received.

Let T be = number of hours in a day receiving 6 calls:

[2 marks]

$T \sim B(10, 0.1462)$

then

~~$P(T \geq 1) = 1.2031$~~ $P(T \geq 1) = 0.5583$

Now let $S \sim B(10, 0.5583)$ be the number of days in 10 with at least 1 hour with 6 calls being received:

Then $P(S \geq 6) = 0.5261$

d) The company also handles claims for broken watch screens.

Assuming that the number of watch claims is independent of phone claims and that the number of watch claims per day can also be modelled by a Poisson distribution, this time with parameter $\lambda = 25$.

What is the standard deviation of the random variable which represents the total number of watch and phone claims per day?

[2 marks]

Let "W be the R.V representing the total number of claims received in a day," then

$\lambda = 25 + 50$
$= 75$

and $W \sim Po(75)$

So standard deviation $= \sqrt{75}$
$= 8.66$

6 A random variable, X, has probability density function

$$f(x) = \begin{cases} -kx(x-3) & 0 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

a) Find the value of k.

[2 marks]

Since, $\int_0^4 -kx(x-3)\,dx = 1$,

$$-k\int_0^4 x^2 - 3x \, dx = 1$$

$$\Rightarrow -k\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^4 = 1$$

$$\Rightarrow -k\left[\left(\frac{64}{3} - \frac{48}{2}\right) - 0\right] = 1$$

$$\Rightarrow -k \cdot \frac{-8}{3} = 1$$

$$\Rightarrow k = \frac{3}{8}$$

Hence the probability density function is

$$f(x) = \begin{cases} -\frac{3}{8}x(x-3) & 0 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$

b) Is the median of $f(x)$ greater than the mean of $f(x)$?

Fully justify your answer.

[3 marks]

Median:

$$\int_0^M -\frac{3}{8}x(x-3)\,dx = \frac{1}{2}$$

$$\Rightarrow -\frac{3}{8}\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^M = \frac{1}{2}$$

$$\Rightarrow -\frac{M^3}{8} + \frac{9M^2}{16} = \frac{1}{2}$$

$$\Rightarrow \frac{M^3}{8} - \frac{9M^2}{16} + \frac{1}{2} = 0$$

So $M = 4.2818$, $M = 1.0817$ or $M = -0.865$

So Median $= 1.0817$ ~~to~~ 4dp

Mean:

$$E(X) = \int_0^4 x f(x)\,dx$$

$$= \int_0^4 -\frac{3}{8}x^2(x-3)\,dx$$

$=$

7 Two examiners A and B are marking maths papers for an awarding organisation.

The number of marking errors per day for each examiner during a 50 day period was recorded.

		Number of Errors				
		0	1	2	3 or more	Total
Examiner	A	5	17	24	4	50
	B	22	10	16	2	50
	Total	27	27	40	6	100

The awarding organisation claim there is an association between examiner and number of errors pay day.

a) Complete the table of expected values

	0	1	2	3 or more
A	13.5	13.5	20	3
B	13.5	13.5	20	3

[1 marks]

b) Test the awarding organisation's claim, at the 1 % significance level.

[5 marks]

Let H_0: There is no association between the number of errors and the examiner

H_1: There is an association between the number of errors and the examiner.

As the E_i in the 3+ column are <5 we need to apply Yate's correction.

Hence, merging the last two columns we get

Observed:

	0	1	2+
A	5	17	28
B	22	10	18

Expected

	0	1	2+
A	13.5	13.5	23
B	13.5	13.5	23

$$\text{dofs} = (3-1)\times(2-1)$$
$$= 3$$

$\chi^{2\,CV}$, for 2 dofs at 1% significance is 9.210

$$\chi^2 \text{ value} = \sum_i \frac{(O_i - E_i)^2}{E_i}$$
$$= 14.6924$$

But, $14.6924 > 9.216$, so there is sufficient evidence to reject H_0.

There is sufficient evidence to conclude there is an association between the number of errors made and the examiner

c) By considering observed and expected frequencies, interpret in context the association between examiner and the number of errors made per day.

[2 mark]

Chi Squared Contributions

	0	1	2+
A	5.3518	0.9074	1.0869
B	5.3518	0.9074	1.0869

Largest source of association is in the 0 errors column. Examiner A makes less than expected 0 error days.

8 The discrete random variable X has probability distribution

x	1	2	3	4
$P(X = x)$	0.3	a	0.25	b

a) Given that $E(X) = 2.45$ and $Var(X) = 1.3475$ find a and b.

[3 marks]

$E(X) = 2.45$ means

$2.45 = 1 \times 0.3 + 2 \times a + 0.25 \times 3 + 4 \times b$

$\Rightarrow \quad 2a + 4b = 1.4 \quad (1)$

Now $E(X^2) = 2.55 + 4a + 16b$, so

$Var(X) = 1.3475$ means that

$1.3475 = 2.55 + 4a + 16b - (2.45)^2$

$\Rightarrow \quad 4.8 = 4a + 16b \quad (2)$

Solving (1) and (2)

$4.8 = 2(1.4 - 4b)$

so $\quad 8b = 2$

$b = 0.25$

and $\quad 1.4 - 0.25 \times 4 = 2a \quad \Rightarrow a = 0.2$

b) Given that the random variable Y is defined to be $Y = 2X + 3$, find

(i) $\mathrm{E}(X)$

$2 \times 2.45 + 3 = 7.9$

(ii) $\mathrm{Var}(X)$

$2^2 \, \mathrm{Var}(x) = 5.39$

[2 marks]

c) The random variable Z has a discrete uniform distribution where $z = 1, 2, \ldots, 10$.

Prove that $\mathrm{E}(Z) = \dfrac{n-1}{2}$ and $\mathrm{Var}(Z) = \dfrac{n^2-1}{12}$.

[3 marks]

↳ Z has a discrete uniform distribution,
then $P(X = x) = \frac{1}{n}$ for $z = 1, 2, \ldots, n$

$$E(X) = \sum_{i=1}^{n} P(X = x)$$

$$= \sum_{i=1}^{n} x_i \frac{1}{n}$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$= \frac{1}{n} \frac{n(n+1)}{2}$$

$$= \frac{n+1}{2}$$

$$E(X^2) = \sum_{x=1}^{n} x^2 P(X=x)$$

$$= \frac{1}{n} \sum x^2$$

$$= \frac{1}{n} \frac{1}{6} n(n+1)(2n+1)$$

$$= \frac{1}{6}(n+1)(2n+1)$$

So $Var(X) = E(X^2) - (E(X))^2$

$$= \frac{1}{6}(n+1)(2n+1) - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(n+1)[2(2n+1) - 3(n+1)]}{12}$$

$$= \frac{n^2-1}{12}$$

d) Given that Z is a discrete distribution with parameter $n = 85a$ where a is the value found in (a), find $\mathrm{E}(X) + \mathrm{E}(Y) + \mathrm{E}(Z)$.

[2 marks]

$85 \times 0.2 = 17 \quad \Rightarrow E(Z) = \frac{17+1}{2} = 9$

Hence, $E(X) + E(Y) + E(Z) = 2.45 + 7.9 + 9$

$= 19.35$